1.3 Shifting, Reflecting, and Stretching Graphs

**Standard:** F-IF #7 – Graph functions and show key features of the graph, by hand.

**Essential Question:** How do shifts, reflections, and stretches affect parent functions?

**Daily Question:** Sketch each function below, by hand, without the use of your notes.

1. \( f(x) = x^2 \)
2. \( f(x) = x^3 \)
3. \( f(x) = |x| \)
4. \( f(x) = \sqrt{x} \)
5. \( f(x) = e^x \)
6. \( f(x) = \log(x) \)
7. \( f(x) = \ln(x) \)

**Critical Point** – our point of reference on a graph which allows us to easily reconstruct the graph given any shift(s), reflection(s), and/or stretch.

**NOW,** plot and label the critical point for each function above.

**Directions:** Sketch a graph of the following functions using your calculator and describe the change from the parent functions below. Plot and label the critical point of each function.

1. \( f(x) = x^2 \)
2. \( f(x) = x^3 \)

Describe the change:

(a) \( \uparrow \) \ 2

(b) \( \uparrow \) \( 3 \)

(c) \( \uparrow \) \( 2 \)
Directions: Sketch a graph of the following functions WITHOUT the use of a calculator. Describe the change from the parent function. Plot and label the critical point of each function.

1. \( f(x) = (x - 1)^3 - 3 \)
   \[ (1, -3) \]
   \[ 3 \]

2. \( f(x) = (x + 5)^2 + 1 \)
   \[ (-5, 1) \]
   \[ 5 \]

Directions: Sketch a graph of the following functions using your calculator and describe the change from the parent functions below. Plot and label the critical point of each function.

1. \( f(x) = |x| \)
   \[ (0, 0) \]

2. \( f(x) = |x + 3| - 2 \)
   \[ (-3, -2) \]
   \[ 2 \]

3. \( f(x) = -|x - 4| \)
   \[ (4, 0) \]
   \[ 4 \]

4. \( f(x) = \sqrt{x} \)
   \[ (0, 0) \]

5. \( f(x) = -\sqrt{x} \)
   \[ (0, 0) \]

6. \( f(x) = \sqrt{-x} \)
   \[ (0, 0) \]

Directions: Sketch a graph of the following functions WITHOUT the use of a calculator. Describe the change from the parent function. Plot and label the critical point of each function.

1. \( f(x) = -|x| + 1 \)
   \[ (0, 1) \]
   \[ 1 \]

2. \( f(x) = \sqrt{-x} - 5 \)
   \[ (0, -5) \]
   \[ 5 \]

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Description of Changes:

1. \( f(x) = (x - 1)^3 - 3 \) reflects over \( y \)-axis.
2. \( f(x) = (x + 5)^2 + 1 \) stretches vertically.
3. \( f(x) = |x + 3| - 2 \) shifts to the left by 3 units and down by 2 units.
4. \( f(x) = -|x - 4| \) shifts to the right by 4 units.
5. \( f(x) = \sqrt{x} \) reflects over \( x \)-axis.
6. \( f(x) = -\sqrt{x} \) reflects over \( y \)-axis.
7. \( f(x) = \sqrt{-x} \) reflects over \( y \)-axis and shifts down by 5 units.
Directions: Sketch a graph of the following functions using your calculator and describe the change from the parent functions below. Plot and label the critical point of each function.

1. \( f(x) = x^2 \)

\[ \begin{array}{c}
\text{Describe the change:} \\
\text{SKINNIER} \\
\text{(VERT. STRETCH)}
\end{array} \]

2. \( f(x) = x^2 \)

\[ \begin{array}{c}
\text{Describe the change:} \\
\text{SKINNIER} \\
\text{(HORIZ. COMPRESSION)}
\end{array} \]

3. \( f(x) = \sqrt{-x - 3} \)

\[ \begin{array}{c}
\text{Reflected over} \\
y-axis
\end{array} \]

Directions: Sketch a graph of the following functions WITHOUT the use of a calculator. Describe the change from the parent function. Plot and label the critical point of each function.

1. \( f(x) = \frac{1}{2} x^3 \)

\[ \begin{array}{c}
\text{WIDER} \\
\text{(VERT. COMPRESSION)}
\end{array} \]

2. \( f(x) = (2x)^2 \)

\[ \begin{array}{c}
\text{SKINNIER} \\
\text{(HORIZ. COMPRESSION)}
\end{array} \]

3. \( f(x) = \sqrt{-x + 2} - 1 \)

\[ \begin{array}{c}
\text{Reflected over} \\
x-axis + y-axis
\end{array} \]

(2, 0)

\( x = 2 \)
Transformation rules

Left and right:
- Set inside equal to 0 + solve

Right and left:
- c outside moves ↑
- c outside moves ↓

Reflect over y-axis:
negative inside of "parenthesis"

Reflect over x-axis:
negative outside of function

Horizontal stretch/Vertical compression:
the coefficient "c" where |c| < 1

Horizontal compression/Vertical stretch:
the coefficient "c" where |c| > 1

Directions: Sketch a graph the following functions WITHOUT the use of a calculator. Label each critical point.

1. \( a(x) = -(x + 3)^3 + 2 \)
2. \( b(x) = -\sqrt{x - 1} + 4 \)
3. \( c(x) = -4x^2 + 1 \)

4. \( d(x) = -\frac{1}{2}|x + 3| - 2 \)
5. \( e(x) = \sqrt{-x - 4} - 3 \)
6. \( f(x) = -(x + 1)^3 + 7 \)
Directions: Write the equation of each function that represents the given transformations.

1. Parent function: \( f(x) = x^3 \); shifted left 2; shifted up 4; reflected over the x-axis
   \[
   f(x) = -\left(x + 2\right)^3 + 4
   \]

2. Parent function: \( f(x) = \sqrt{x} \); shifted right 5; shifted up 1; reflected over the y-axis
   \[
   f(x) = \sqrt{-x + 5} + 1
   \]

3. Parent function: \( f(x) = |x| \); the vertex is located at (3, 8); the function opens down
   \[
   f(x) = -|x - 3| + 8
   \]

4. Parent function: \( f(x) = x^2 \); the vertex is located at (-6,4); the function opens up
   \[
   f(x) = (x + 6)^2 + 4
   \]

Exponential & Logarithmic Functions

Directions: Sketch a graph of the following functions using your calculator and describe the change from the parent functions below. Plot and label the critical point of each function.

1. \( f(x) = \log(x) \)

   \[
   \text{Describe the change: } \quad \bigcirc \quad 3
   \]

2. \( f(x) = \log(x - 3) \)

   \[
   \text{Describe the change: } \quad \downarrow \quad 3
   \]

3. \( f(x) = -\log(x) \)

   \[
   \text{Describe the change: } \quad \text{Reflected over } y\text{-axis}
   \]

4. \( f(x) = \log(-x) \)

   \[
   \text{Describe the change: } \quad \text{Reflected over } y\text{-axis}
   \]
2. \( f(x) = e^x \)

\[ (0,1) \]

\[ \rightarrow \]

Describe the change:

\[ (3,1) \]

\[ \rightarrow \]

\( f(x) = e^{x-3} \)

[Image]

Describe the change:

\[ (0,-2) \]

\[ \rightarrow \]

\[ f(x) = e^x - 3 \]

[Image]

Describe the change:

Things to REMEMBER about Exponential & Logarithmic Graphs:

- Always REFLECT before you SHIFT
- When SHIFTING, remember to SHIFT THE ASYMPTOTE as well

Directions: Sketch a graph the following functions WITHOUT the use of a calculator. Label each critical point.

1. \( a(x) = -\log(x + 2) \)

\[ (-1,0) \]

\[ \rightarrow \]

2. \( b(x) = e^{-x+1} - 4 \)

\[ (-1,-2) \]

\[ \rightarrow \]

3. \( c(x) = \ln(x) - 3 \)

\[ (1,-2) \]

\[ \rightarrow \]

4. \( d(x) = e^x + 3 \)

\[ (0,4) \]

\[ \rightarrow \]

5. \( e(x) = \log(-x - 1) \)

\[ (-1,0) \]

\[ \rightarrow \]

6. \( f(x) = -e^{-x-3} + 2 \)

\[ (-3,1) \]

\[ \rightarrow \]
**Directions:** Use the graph of $f$ to sketch each transformation below.

1. $y = f(x) - 1$
   - Down 1

2. $y = f(x + 1)$
   - Left 1

3. $y = -f(x)$
   - Reflected over x-axis

4. $y = f(-x)$
   - Reflected over y-axis

5. $y = 2f(x) + 1$
   - y-values x 2
   - Up 1

6. $y = -f(x + 2) + 3$
   - Reflected over x-axis
   - Left 2
   - Up 3

**Homework**

**Part 1:** Sketch a graph of the following functions without the use of a calculator. Label the critical point of each function.

1. $a(x) = 4 - x^2$
   - $(0, 4)$

2. $b(x) = 4(-x - 4)^3 - 1$
   - $-x - 4 < 0$
   - $x = -4$

3. $c(x) = -\frac{1}{3}|x + 6| - 2$
   - $g(x) = f(x + 2)$
   - $h(x) = \frac{1}{2}f(x)$

**Part 2:** DESCRIBE the graphs of $g$ and $h$ relative to the graph of $f$.

1. $f(x) = x^3 - 3x^2$
   - $g(x) = f(x + 2)$
   - $h(x) = \frac{1}{2}f(x)$
   - $g(x) \rightarrow$ LEFT 2
   - $h(x) \rightarrow$ WIDER

2. $f(x) = x^3 - 3x^2 + 2$
   - $g(x) = -f(x)$
   - $h(x) = f(-x)$
   - $g(x) \rightarrow$ Reflected over x-axis
   - $h(x) \rightarrow$ Reflected over y-axis
Part 3: Graph each function below WITHOUT the use of a calculator. Label the critical point of each function.

1. \( a(x) = -\log(x + 2) - 4 \)

\[ (-1, -4) \]

2. \( b(x) = e^{-x^2} + 3 \)

\[ (1, 2) \]

3. \( c(x) = \log(x - 1) + 3 \)

\[ (2, 3) \]

4. \( d(x) = 2\ln(x) - 2 \)

\[ (-1, -2) \]

Part 4: Use the graph of \( g \) (below) to sketch each transformation.

1. \( y = g(x) + 2 \)

\[ \text{Up 2} \]

2. \( y = g(x + 3) - 1 \)

\[ \text{Left 3, Down 1} \]

3. \( y = -g(x - 2) \)

\[ \text{Reflected over x-axis, Right 2} \]

4. \( y = g(-x + 1) - 4 \)

\[ \text{Reflected over y-axis, Right 1, Down 4} \]

5. \( y = 3g(x) - 6 \)

\[ \text{y-values x 3, Down 6} \]